ELECTRONICS 101 · GATE SERIES

Signals and Systems

for GATE EC / EE / IN and ESE

FREE SAMPLE · CHAPTER 1

Basics of Signals

Definitions, solved examples and GATE-style questions,
typeset from detailed handwritten class notes.

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Chapter 1

Basics of Signals

1.1 Course outline

1. Basics of signals
2. Systems
3. Continuous time Fourier series (CTFS)
4. Continuous time Fourier transform (CTFT)
5. Sampling of C.T. signals
6. Laplace transforms
7. Z-transform
8. DTFT, DTFS, DFT, FFT, DCT $\quad \rightarrow$ ESE
9. Filters

1.2 Signals and functions

Definition 1.1 (Signal)**.** A signal is a mathematical representation of a physical phenomenon which contains some information.

For example, taking the physical phenomenon to be an AC voltage, the mathematical representation is

$$x(t) = A\cos\omega_0 t,$$

where x is the signal, A its amplitude and ω_0 its frequency.

Definition 1.2 (Function)**.** A function is a dependent variable (DV) which depends upon one or more independent variables (IDV).

For example, in $x(t) = t^2 + 1$ the quantity $x(t)$ is the dependent variable and t is the independent variable.

Example 1.3. Identify the dependent and independent variables.

1. $x(t)$: $\quad x(t) \rightarrow \text{DV}, \quad t \rightarrow \text{IDV}.$
2. $x(\lambda, t)$: $\quad x(\lambda, t) \rightarrow \text{DV}, \quad \lambda, t \rightarrow \text{IDV}.$
3. $x(t) = A\sin\omega_0 t$ is both a signal and a function.

Example 1.4. $x(t) = |t|$ — this is a function, but is it a signal?

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1.2.1 Difference between a signal and a function

Signal	Function
1. Contains information.	1. May or may not.
2. Single valued, i.e. $t_1 \longrightarrow x(t_1)$.	2. Multi valued, i.e. t_1 maps to both $x(t_1) = \lambda$ and $x(t_1) = \emptyset$.

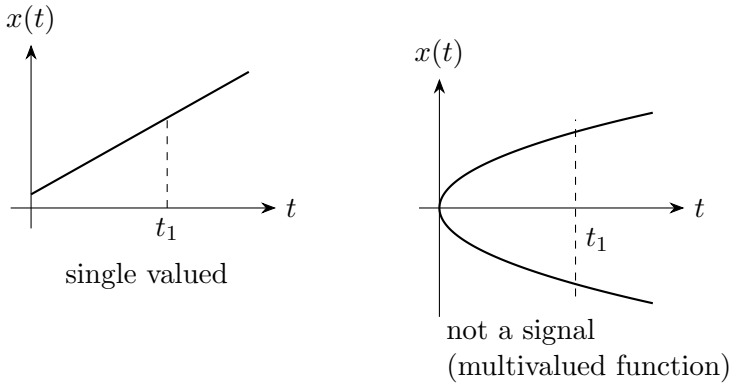


Figure 1.1: A single-valued map qualifies as a signal; a multivalued one cannot.

Key Point 1.5. All signals become functions, but all functions need not be signals.

For example, $x(t) = A\cos\omega_0 t$ is a signal, and therefore also a function; whereas $x(t) = |t|$ is a function but not a signal.

Example 1.6. Identify which of the following are *not* qualified as a signal.

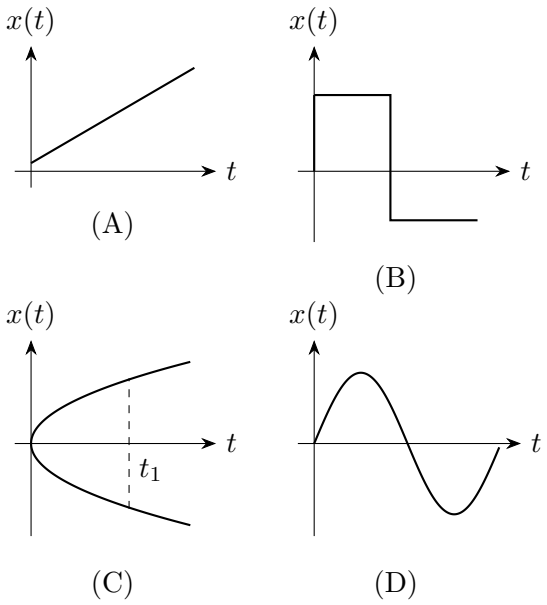


Figure 1.2: Which of these is not a signal?

Answer: (C). At t_1 it is multivalued, so it is not qualified as a signal.

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1.3 Important notations

1. $x(t)$ — DV: $x(t)$; IDV: t = continuous time. Hence a *continuous time domain signal* (CTDS).
2. $x[n]$ — DV: $x[n]$; IDV: n = discrete time. Hence a *discrete time domain signal* (DTDS).
3. $X(\omega)$ or $X(f)$ — DV: $X(\omega)$ or $X(f)$; IDV: ω or f. Hence a *continuous frequency domain signal* (CFDS).
4. $X(n\omega_0)$ — DV: $X(n\omega_0)$, $n \in \mathbb{I}$; IDV: $n\omega_0$ = discrete frequency. Hence a *discrete frequency domain signal* (DFDS).

Here ω is the angular frequency in rad/sec and f is the linear frequency in Hz, related by

$$\omega = 2\pi f.$$

From		Transform	To	
$x(t)$	(CTDS)	C.T.F.T.	$X(\omega)$	(CFDS)
$x(t)$	(CTDS)	C.T.F.S.	$X(n\omega_0)$	(DFDS)
$x[n]$	(DTDS)	DTFT	$X(\omega)$	(CFDS)
$x[n]$	(DTDS)	—	$X(n\omega_0)$	(DFDS)

1.4 Classification of signals

1.4.1 Continuous time and discrete time signals

A signal is continuous (or continuous in amplitude) if

$$\lim_{t \to a} f(t) = f(a) = \text{finite}.$$

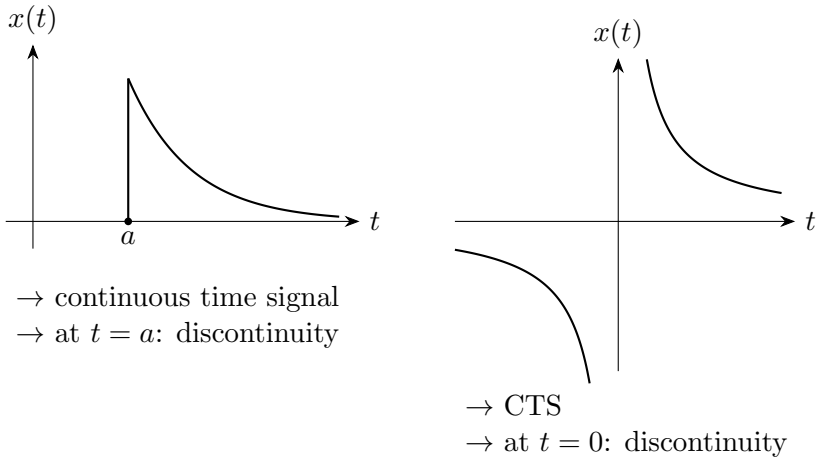


Figure 1.3: Continuous time signals with a discontinuity.

1.4.2 Discrete time signals

Discrete time signals are obtained from continuous time signals by a process called *uniform sampling*. The continuous time signal $x(t)$ is passed through a switch closing at the instants $t = nT_s$, that is at $t = 0,\ T_s,\ 2T_s,\ \ldots$, producing the sampled signal $x(nT_s)$ for $n = 0, 1, 2, \ldots$

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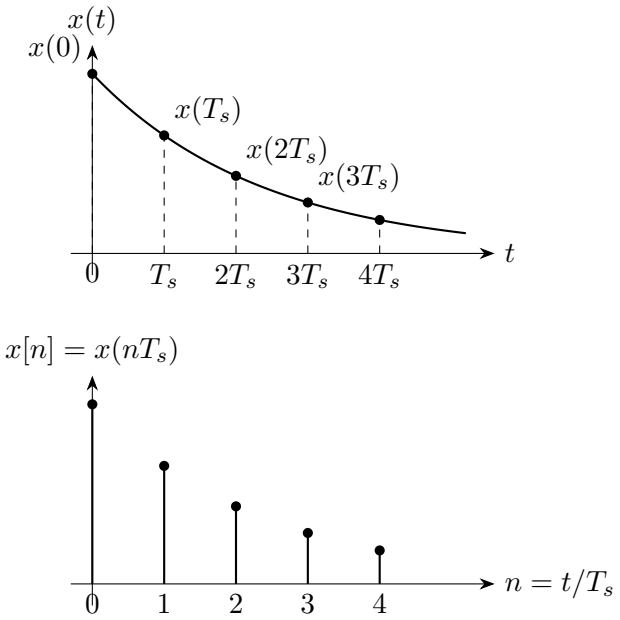


Figure 1.4: Uniform sampling of a continuous time signal. T_s is the sampling interval.

1.4.3 Steps to convert a CTS into a DTS

Given $x(t)$ and T_s, with $f_s = 1/T_s$ the sampling frequency in Hz, and $x(t)$ defined on $a \leq t \leq b$:

Derivation 1.7. Put $t = nT_s$. Let $x(t) = f(t)$ on $a \leq t \leq b$. Then

$$x(nT_s) = f(nT_s); \qquad a \leq nT_s \leq b,$$

and dividing the range through by T_s,

$$x[n] = f(nT_s); \qquad \frac{a}{T_s} \leq n \leq \frac{b}{T_s}.$$

Example 1.8. $x(t) = \cos(100\pi t)$. Convert into a DTS when the sampling frequency is 4 Hz.

Solution. $f_s = 4\,\text{Hz}$, so $T_s = \dfrac{1}{f_s} = \dfrac{1}{4} = 0.25\,\text{sec}$. Then

$$x[n] = x(t)|_{t=nT_s} = \cos(100\pi \times n \times 0.25),$$

$$\boxed{x[n] = \cos(25\pi n)}$$

Example 1.9. $x(t) = 1 - |t|$, $-1 \leq t \leq 1$. Obtain the corresponding discrete signal if the sampling frequency is $f_s = 0.5\,\text{Hz}$.

Solution. $T_s = \dfrac{1}{f_s} = \dfrac{1}{0.5} = 2\,\text{sec}$, so $nT_s = 2n$ and

$$x[n] = x(t)|_{t=nT_s=2n} = 1 - |2n|; \qquad -1 \leq 2n \leq 1,$$

$$x[n] = 1 - |2n|; \qquad -\tfrac{1}{2} \leq n \leq \tfrac{1}{2}.$$

Only $n = 0$ exists, because n must be an integer and cannot be a fraction. Therefore

$$x[n] = \begin{cases} x(0), & n = 0, \\ 0, & \text{else.} \end{cases}$$

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Example 1.10. $x(t) = 1 - |t|$, $-4 \leq t \leq 4$, $f_s = 0.5\,\text{Hz}$. Find $x[n]$.

Solution. $T_s = \dfrac{1}{f_s} = 2$ and $nT_s = 2n$, so

$$x(nT_s) = x(2n); \qquad -4 \leq 2n \leq 4,$$

$$x[n] = 1 - |2n|; \qquad -2 \leq n \leq 2.$$

Example 1.11. $x(t) = 5e^{-t/2}$, $f_s = 10\,\text{Hz}$.

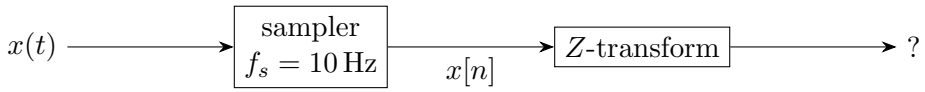


Figure 1.5: Sampling followed by the Z-transform.

Solution. $T_s = \dfrac{1}{f_s} = \dfrac{1}{10}$ sec, so

$$x[n] = x(t)|_{t=nT_s} = 5e^{-\frac{n}{2\times 10}} = 5e^{-n/20},$$

$$x[n] = 5e^{-(0.05)n}.$$

1.5 Operations on continuous time signals

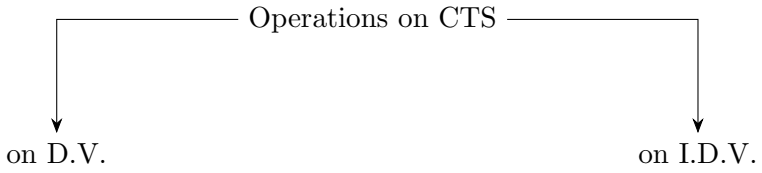


Figure 1.6: Operations act either on the dependent or the independent variable.

1.5.1 Operations on the dependent variable

1. Amplitude scaling

If $x(t)$ is given, what is $A \cdot x(t)$? Every vertical axis parameter is multiplied by A, where A may be real, imaginary or complex.

Example 1.12. For the triangular signal below, find $y(t) = 2 \cdot x(t)$.

Key Point 1.13. Amplitude scaling changes the slope by A times, and the area is also scaled by a factor of A.

2. Amplitude reversal

If $x(t)$ is a given signal, then

$$\boxed{y(t) = -\,x(+t)}$$

Example 1.14. For the signal $x(t)$ below, sketch $y(t) = -\frac{1}{2}x(t)$.

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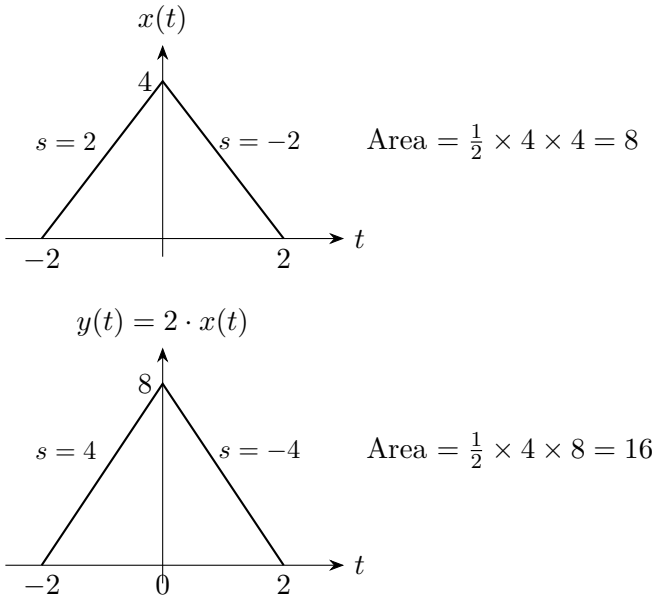


Figure 1.7: Amplitude scaling by $A = 2$.

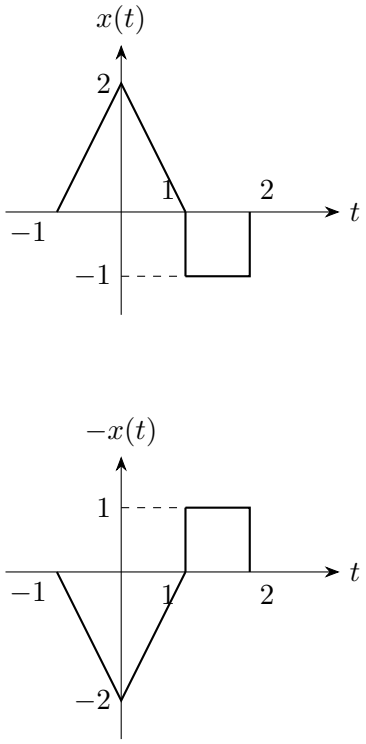


Figure 1.8: Amplitude reversal: the signal is flipped about the horizontal axis.

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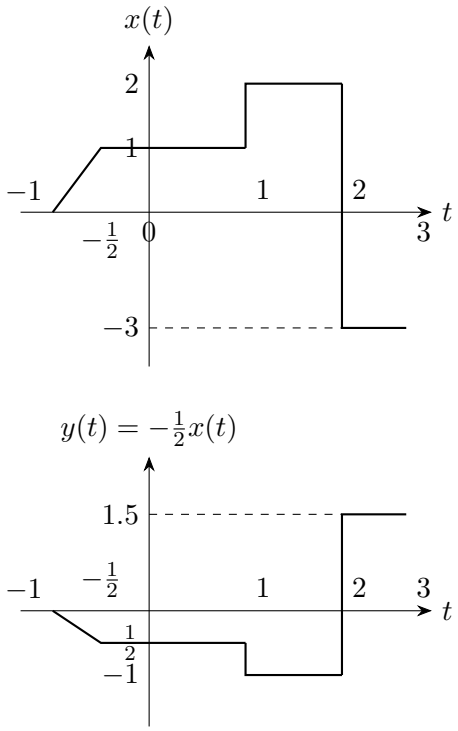


Figure 1.9: Amplitude scaling by $A = -\frac{1}{2}$.

3. Modulus operation

If $x(t)$ is given, find $|x(t)|$.

1. Take the graph as it is above the horizontal axis.
2. Take the mirror image of the graph below the horizontal axis.

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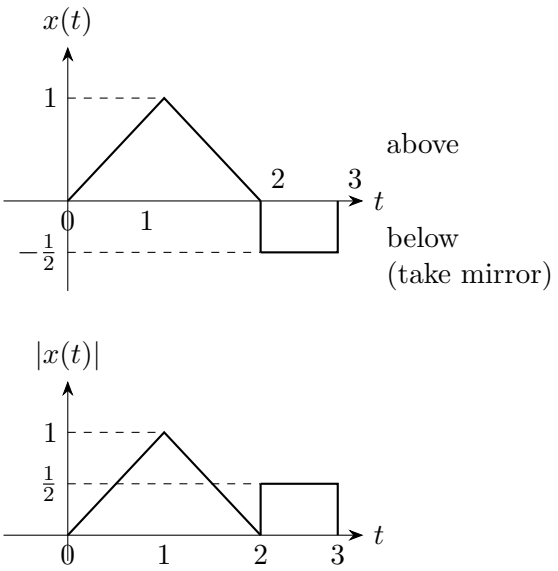


Figure 1.10: The modulus operation reflects the negative part upward.

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4. Addition / subtraction of DC

If $x(t)$ is given, what is $x(t) \pm A$, where A is a constant over $-\infty \leq t \leq \infty$? The whole signal is shifted vertically by A; the shape and the slopes are untouched.

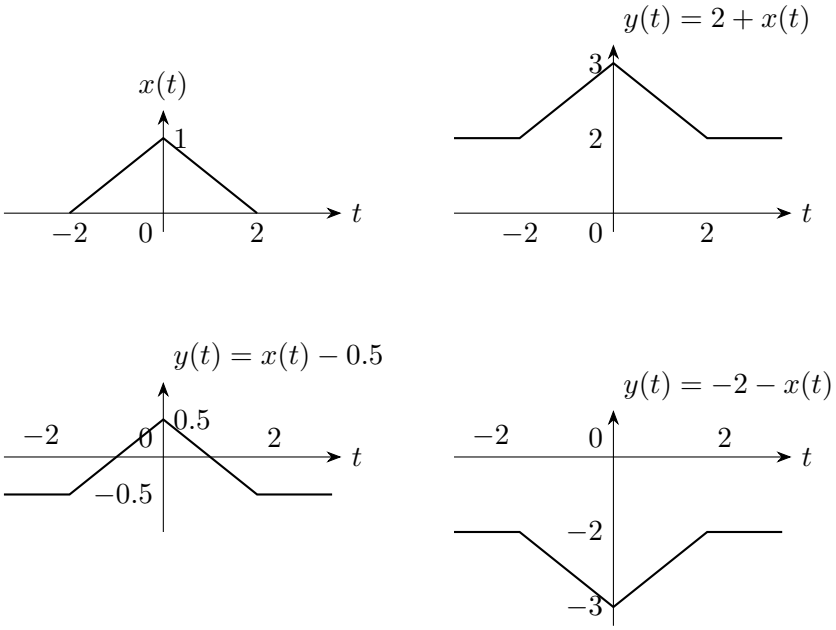


Figure 1.11: Adding or subtracting a DC level shifts the signal vertically.

1.5.2 Operations on the independent variable

1. Time shifting

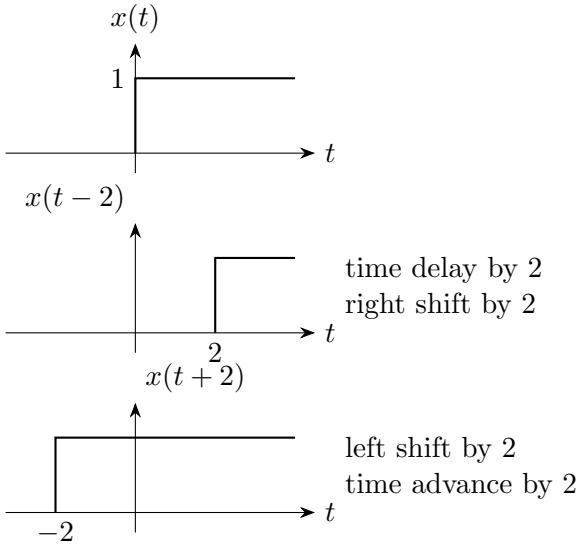


Figure 1.12: Time shifting. $x(t-2)$ delays, $x(t+2)$ advances.

2. Time scaling

Key Point 1.15. For $x(at)$: if $a > 1$ the graph is compressed by a; if $a < 1$ the graph is expanded by a.

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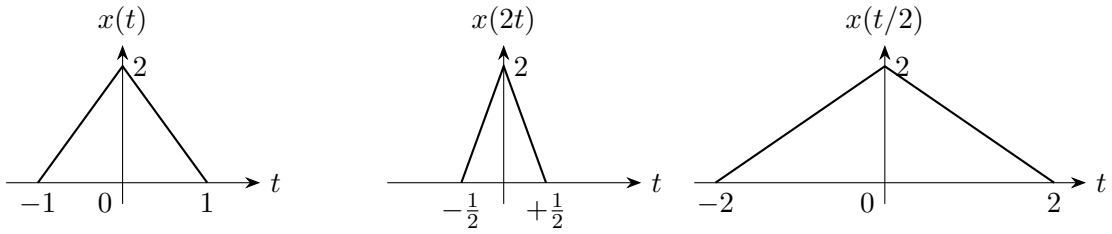


Figure 1.13: Time scaling.

Example 1.16. Plot $x(t+2)$ versus $t+2$.

Solution. The given plot is $x(t)$ versus t. The plot of $x(t+2)$ versus $t+2$ is exactly the same, because the time axis changes along with the signal.

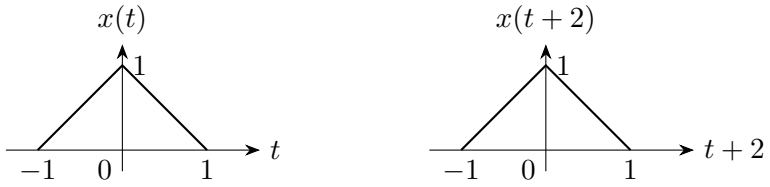


Figure 1.14: Plotting $x(t+2)$ against $t+2$ reproduces the original plot.

3. Time reversal

To obtain $x(-t)$, take the mirror image of the right half to the left and of the left half to the right, with respect to the vertical axis.

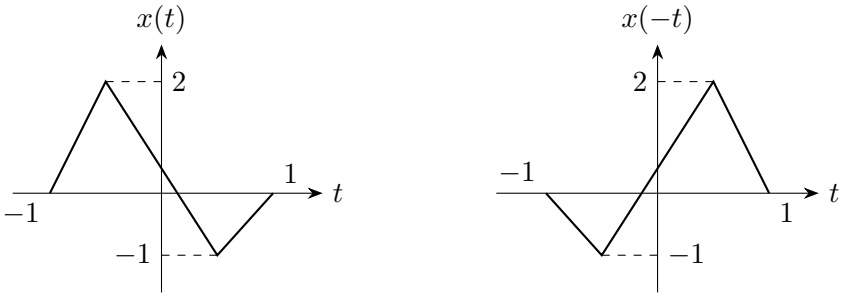


Figure 1.15: Time reversal is a reflection about the vertical axis.

4. Modulus operation on t

Given the plot of $x(t)$, to find $x(|t|)$:

1. Ignore the graph to the left of the vertical axis.
2. Take the mirror image of the graph on the right side.

1.5.3 Mixed operations

Given $x(t)$, find $x(-at+b)$.

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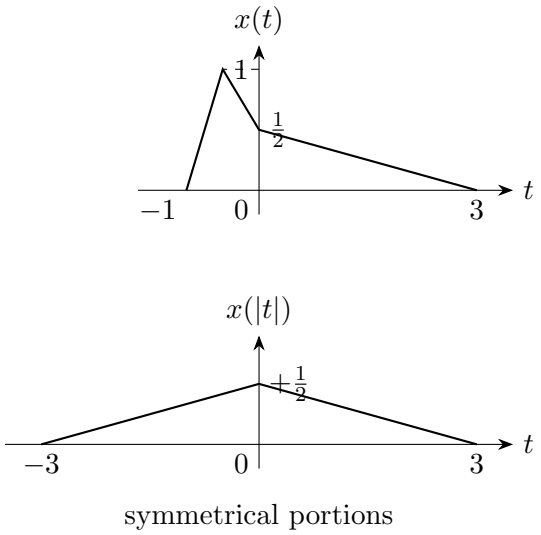


Figure 1.16: The modulus operation on t keeps the right half and mirrors it.

Derivation 1.17. *Method 1 — the natural order.*

$$x(t) \xrightarrow{\text{shift by } b} x(t+b) \xrightarrow{\text{scaling by } a} x(at+b) \xrightarrow{\text{time reversal}} x(-at+b)$$

that is, time shift → time scaling → time reversal.

Method 2.

$$x(t) \xrightarrow{\text{time scaling}} x(at) \xrightarrow{\text{time reversal}} x(-at) \xrightarrow{\text{time shift}} x\big(-a\left(t-\tfrac{b}{a}\right)\big) = x(-at+b)$$

In the same way $x(t) \to x(-t) \to x(-(t-b)) = x(-t+b)$.

Example 1.18. For the signal below, find $x(-2t+3)$.

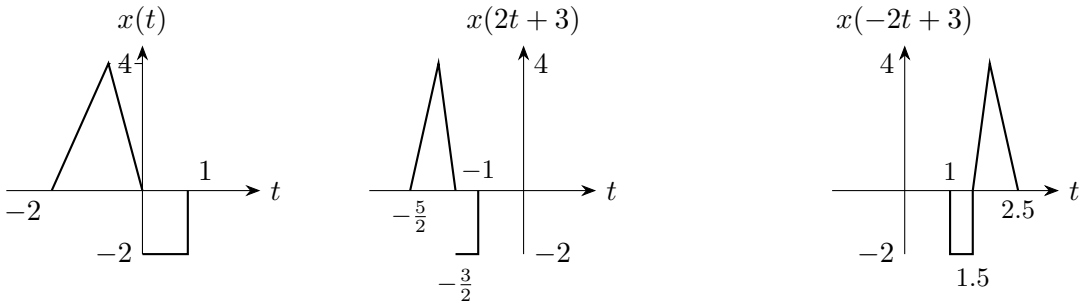


Figure 1.17: $x(t) \to x(2t+3) \to x(-2t+3)$.

Note. Source page 14 sketches the first step shifted to the *right* ($x(t-3)$), while page 15 continues from $x(t+3)$. The chain above follows page 15, which is the one consistent with the final answer.

The same example by Method 2. The notes define two methods but work this example only by the first. Method 2 scales first, reverses, and shifts last:

$$x(t) \xrightarrow{\text{time scaling}} x(2t) \xrightarrow{\text{time reversal}} x(-2t) \xrightarrow{\text{shift by } 3/2} x\big(-2\left(t-\tfrac{3}{2}\right)\big) = x(-2t+3)$$

Note the shift in the last step is $b/a = 3/2$, not 3 — the scaling has already compressed the axis, so the shift must be compressed with it. This is the step Method 1 avoids by shifting first.

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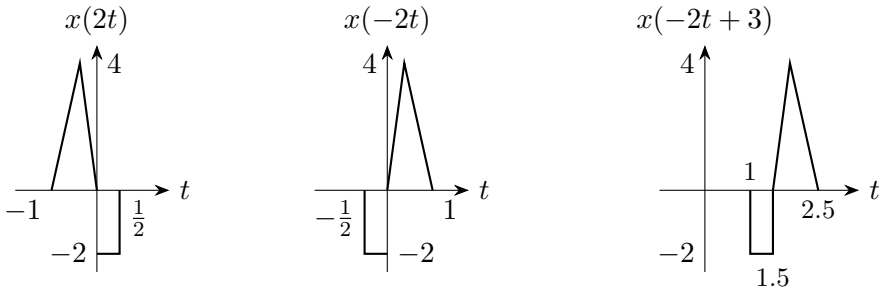


Figure 1.18: Method 2 on the same signal: scale, reverse, then shift by b/a. The result agrees with Figure 1.17.

Key Point 1.19. Both methods must land on the same signal. Working an awkward problem twice — once shifting first, once scaling first — is the cheapest available check on the answer.

Example 1.20. Plot $x\left(\dfrac{1-2t}{3}\right)$.

Solution. Writing the argument as $-\frac{2}{3}t + \frac{1}{3}$ and comparing with $x(-at+b)$ gives $a = \frac{2}{3}$, $b = \frac{1}{3}$, so the chain is

$$x(t+1/3) \;\to\; x\left(\tfrac{2}{3}t + \tfrac{1}{3}\right) \;\to\; x\left(-\tfrac{2}{3}t + \tfrac{1}{3}\right).$$

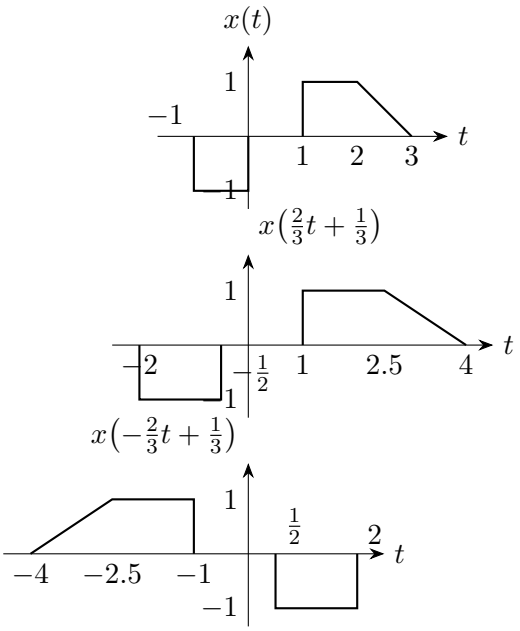


Figure 1.19: Building $x\left(\frac{1-2t}{3}\right)$ by scaling then reversal.

1.6 Addition and subtraction of continuous time signals

For non-sinusoidal continuous time signals:

1. Always travel from left to right.
2. At any point, hold the position while travelling left to right.
3. Where one signal steps up and the other steps down, take the net effect at that point.

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Key Point 1.21. Over a range in which one signal has slope m and the other is constant ($s = 0$), the result is an *end point addition* plus a *slope addition* — which amounts to a level shift: a segment from a to b added to a constant k becomes a segment from $a + k$ to $b + k$.

Over a range in which *both* signals have slopes m_1 and m_2, it is again end point addition plus slope addition, but it is *not* a level shift.

Note also that subtraction is addition of the reversed signal:

$$y(t) = x_1(t) + x_2(t), \qquad y(t) = x_1(t) - x_2(t) = x_1(t) + (-x_2(t))\,.$$

Example 1.22. Plot $x_1(t) + x_2(t)$ for the unit steps below.

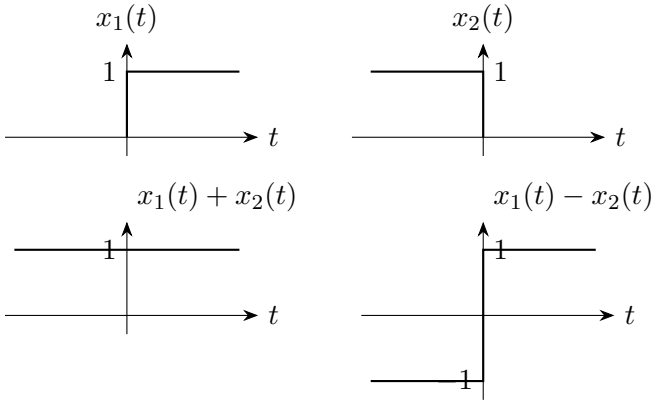


Figure 1.20: Sum and difference of two complementary steps.

Example 1.23. Plot $x_1(t) + x_2(t)$ for the trapezoid and ramp below.

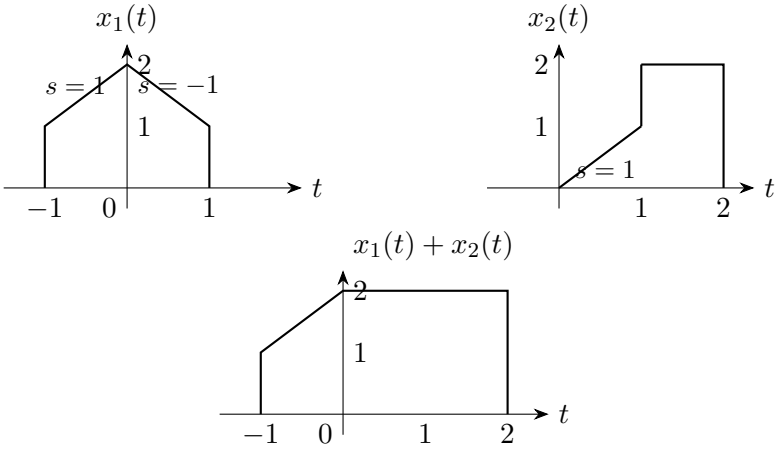


Figure 1.21: Slope addition: the falling slope -1 and the rising slope $+1$ cancel, giving a flat top.

Example 1.24. Plot $x_1(t) + x_2(t)$ where $x_2(t)$ is a rectangle.

Example 1.25. For the signal $x(t)$ below, plot $x(t + 2) + 2\,x(t + 1)$ and $x(t + 2) - 2\,x(t + 1)$.

Here $x(t + 2)$ is a left shift by 2, and $2\,x(t + 1)$ is a left shift by 1 together with an amplitude scaling by 2.

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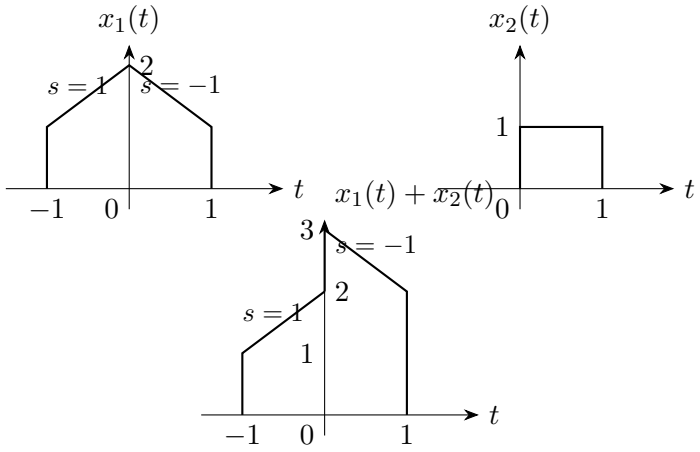


Figure 1.22: End point addition with a jump at $t = 0$.

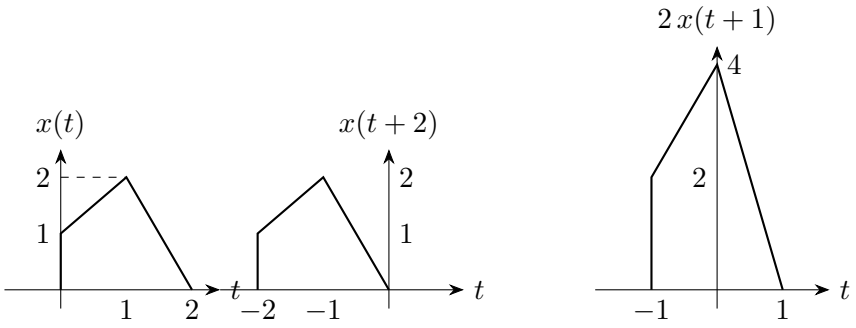


Figure 1.23: The two shifted and scaled components.

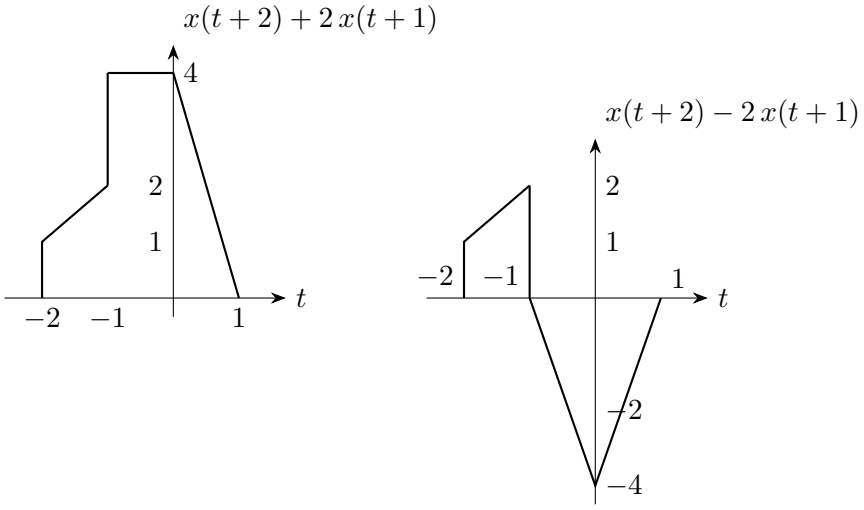


Figure 1.24: Sum and difference of the two components.

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Homework

Plot the following.

1. $y(t) = x(t+1) - 2\,x(t+2)$
2. $y(t) = 2\,x(t+1) - 4\,x(t)$
3. $y(t) = -\frac{1}{2}\,x(t) - \frac{3}{2}\,x(t-1)$
4. $y(t) = x(t+1) - \frac{1}{2}\,x(t-1)$

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1.7 Argument and breakpoint

For $x(-2t+3)$ the *argument* is $-2t+3$, and the *breakpoint* is the value of t at which the argument vanishes:

$$-2t+3=0 \quad \Longrightarrow \quad \boxed{t=\tfrac{3}{2}}$$

Likewise for $x(-2t+1)$ the argument is $-2t+1$ and

$$-2t+1=0 \quad \Longrightarrow \quad \boxed{t=\tfrac{1}{2}}$$

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3. Discrete Time Signals
4. Even and Odd Signals
5. Periodic and Non-Periodic Signals
6. Orthogonal and Non-Orthogonal Signals
7. Energy, Power and NENP Signals
8. Causal, Bounded and Integrable Signals
9. Systems
10. Convolution and CT LTI Systems
11. Convolution and DT LTI Systems
12. Properties of LTI Systems
13. Continuous Time Fourier Series
14. CT Fourier Transform
15. Sampling
16. Laplace Transform
17. Z-Transform
18. DTFT, DFT and FFT
19. Filters

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